Black Holes From Diifferent Perspectives

B.G. Sidharth
Centre for Applicable Mathematics & Computer Sciences
B.M. Birla Science Centre, Adarsh Nagar, Hyderabad - 500063 (India)

Abstract

In this paper we consider black holes from a non general relativistic perspective as also from a microphysical point of view.

1 Introduction

It is generally believed that the concept of Black Holes requires General Relativity for its understanding and interpretation. In this brief note we will show that Black Holes could also be understood without invoking General Relativity at all.

We start by defining a Black Hole as an object at the surface of which, the escape velocity equals the maximum possible velocity in the universe viz., the velocity of light. We next use the well known equation of Keplerian orbits[1],

$$\frac{1}{r} = \frac{GM}{L^2}(1 + ecos\theta) \tag{1}$$

where L, the so called impact parameter is given by, Rc, where R is the point of closest approach, in our case a point on the surface of the object and c is the velocity of approach, in our case the velocity of light.

Choosing $\theta = 0$ and $e \approx 1$, we can deduce from (1)

$$R = \frac{2GM}{c^2} \tag{2}$$

Equation (2) gives the Schwarzchild radius for a Black Hole and can be deduced from the full General Relativistic theeory[2].

We will now use (2) to exhibit Black Holes at three different scales, the micro, the macro and the cosmic scales.

2 Black Holes

Our starting point is the observation that a Planck mass, $10^{-5}gms$ at the Planck length, $10^{-33}cms$ satisfies (2) and, as such is a Schwarzchild Black Hole (Cf.ref.[3]). Infact Rosen has used non-relativistic Quantum Theory to show that such a particle is a mini universe[4].

We next come to stellar scales. It is well known that for an electron gas in a highly dense mass we have[5]

$$K\left(\frac{\bar{M}^{4/3}}{\bar{R}^4} - \frac{\bar{M}^{2/3}}{\bar{R}^2}\right) = K'\frac{\bar{M}^2}{\bar{R}^4} \tag{3}$$

where

$$\left(\frac{K}{K'}\right) = \left(\frac{27\pi}{64\alpha}\right)\left(\frac{\hbar c}{\gamma m_P^2}\right) \approx 10^{40} \tag{4}$$

and

$$\bar{M} = \frac{9\pi}{8}\frac{M}{m_P} \quad \bar{R} = \frac{R}{(\hbar/m_e c)},$$

M is the mass, R the radius of the body, m_P and m_e are the proton and electron masses and $\hbar$ is the reduced Planck Constant. From (3) and (4) it is easy to see that for $\bar{M} < 10^{60}$, there are highly condensed planet sized stars. (Infact these considerations lead to the Chandrasekhar limit in stellar theory). We can also verify that for $\bar{M}$ approaching 10^{60} corresponding to a mass $\sim 10^{36}gms$, or roughly a hundred to a thousand times the solar mass, the radius R gets smaller and smaller and would be $\sim 10^8cms$, so as to satisfy (2) and give a Black Hole in broad agreement with theory.

Finally for the universe as a whole, using only the theory of Newtonian gravitation, it is well known that we can deduce

$$R \sim \frac{GM}{c^2} \tag{5}$$

where this time $R \sim 10^{28}cms$ is the radius of the universe and $M \sim 10^{55}gms$ is the mass of the universe.

Equation (5) suggests that the universe itself is a Black Hole. It is remarkable that if we consider the universe to be a Schwarzchild Black Hole as suggested by (5), the time taken by a ray of light to traverse the universe equals the age of the universe $\sim 10^{17}secs$ as shown elsewhere [6].

3 The Kerr-Newman Formulation for the Electron

It was already noted[7], that a particle with the Planck mass viz. $10^{-5} gms$ could be considered to be a Schwarzchild black hole whose radius is of the order of the Planck length viz., $10^{-33} cms$. One could then ask whether a charged rotating black hole, that is a Kerr-Newman black hole could represent an elementary particle with charge. Indeed the remarkable fact has been well known[2] that the purely classical Kerr-Newman metric does describe the electron, including its purely Quantum Mechanical anomalous gyro magnetic ratio $g = 2$! This could have been construed to be the much sought after unification of General Relativity and Quantum Mechanics, except for the fact that such a Kerr-Newman electron black hole would have a naked singularity. That is, its radius becomes complex:

$$r_+ = \frac{GM}{c^2} + \imath b, b \equiv (\frac{G^2Q^2}{c^8} + a^2 - \frac{G^2M^2}{c^4})^{1/2} \tag{6}$$

where G is the gravitational constant M the mass and $a \equiv L/Mc, L$ being the angular momentum.
Even in the derivation of the above Kerr-Newman metric, Newman has noted[8] the puzzling fact that an imaginary shift of coordinates has to be invoked and that it is this imaginary shift which gives the rotation or spin. From a classical point of view this is inexplicable.
On the other hand it has been pointed out by the author[9] that the Quantum Mechanical coordinate of a Dirac electron is given by

$$x = (c^2 p_1 H^{-1} t + a_1) + \frac{\imath}{2} c\hbar (\alpha_1 - cp_1 H^{-1}) H^{-1}, \tag{7}$$

where a_1 is an arbitrary constant and $c\alpha_1$ is the velocity operator with eigen values $\pm c$.
It has also been noted that for the electron the imaginary parts in (6) and (7) are of the same order, and that this imaginary coordinate was given a physical explanation long ago by Dirac[10]: This has to do with the famous Zitterbewegung. As space time intervals shrink, by Heisenberg's Uncertainty Principle the uncertainty in the momentum - energy values increases. Thus only averages over space time intervals, specifically at the Compton scale, are

physically meaningful. Within the Compton scale we encounter unphysical Zitterbewegung effects which show up as complex (or non-Hermitian) coordinates (Cf.also ref.[11]).
So the unsatisfactory feature of the Kerr-Newman electron black hole can be circumvented with the Quantum Mechanical input that rather than space time points as in classical theory, we need to consider averages over Compton scale space time intervals.
All this pleasingly dovetails with the fact that minumum space time cut offs can be taken consistently with the Lorentz transformation as shown by Snyder a long time ago[12]. In recent years there has ben a return to ideas of discrtee space time including through string theory[13]. Further if the cut off is at the Compton scale (l, τ) then we have a non commutative geometry[13] viz.,

$$[x, y] = 0(l^2), [x, p_x] = \imath\hbar(1 - l^2) \tag{8}$$

and similar equations, which can be shown to directly lead to the Dirac equation. In other words it is this minimum Compton scale space time cut off as in equation (8) which leads to the Dirac matrices, spin and the anomalous gyro magnetic ratio $g = 2$. Indeed it has been noted recently by Ne'eman[14] that such a non commutative geometry provides a rationale for renormalization. Infact this was the motivation for the very early work of Snyder and others in introducing discrete space time.
It must be mentioned that if in (8) terms $\sim l^2$ are neglected, then we recover the usual commutation relations of Quantum theory.

4 Some Experimental Consequences

The question that arises is, are there any experimental consequences of the above formulation[15]:
I. We first observe that the magnetic component of the field of a static electron as a Kerr-Newman black hole is given in the familiar spherical polar coordinates by (Cf.refs.[9, 11]).

$$B_{\hat{r}} = \frac{2ea}{r^3} cos\Theta + 0(\frac{1}{r^4}), B_{\hat{\Theta}} = \frac{easin\Theta}{r^3} + 0(\frac{1}{r^4}), B_{\hat{\phi}} = 0, \tag{9}$$

whereas the electrical part is given by

$$E_{\hat{r}} = \frac{e}{r^2} + 0(\frac{1}{r^3}), E_{\hat{\Theta}} = 0(\frac{1}{r^4}), E_{\hat{\phi}} = 0, \tag{10}$$

A comparison of (9) and (10) shows that there is a magnetic component of shorter range apart from the dipole which is given by the first term on the right in equation (9). We would like to point out that a short range force, the $B^{(3)}$ force, mediated by massive photons has indeed been observed at Cornell and studied over the past few years[16].
On the other hand as the Kerr-Newman charged black hole can be approximated by a solenoid, we have as in the Aharonov-Bohm effect, a negligible magnetic field outdside, but at the same time a real vector potential $\vec{A}$ which would contribute to a shift in phase. Infact this shift in phase is given by (Cf. also ref.[17])

$$\Delta\delta_{\hat{B}} = \frac{e}{\hbar}\oint \vec{A}.\vec{ds} \tag{11}$$

There is also a similar effect due to the electric charge given by

$$\Delta\delta_{\hat{E}} = -\frac{e}{\hbar}\int A_0 dt \tag{12}$$

where A_0 is the usual electro static potential given in (10). In the above Kerr-Newman formulation, $(\vec{A}, A_0)$ of (11) and (12) are given by (Cf.refs.[9, 11])

$$A_\sigma = \frac{1}{2}(\eta^{\mu v} h_{\mu v}), \sigma, \tag{13}$$

From (13) it can be seen that

$$\vec{A} \sim \frac{1}{c}A_0 \tag{14}$$

Substitution of (14) in (11) then gives us the contribution of the shift in phase due to the magnetic field.
II. Let us now consider some imprints of discrete space time, as discussed in section 1.
First we consider the case of the neutral pion. As is known, this pion decays into an electron and a positron. Could we think of it as an electron-positron bound state also[11, 18, 19, 20]? In this case we have,

$$\frac{mv^2}{r} = \frac{e^2}{r^2} \tag{15}$$

Consistently with the above formulation, if we take $v = c$ from (15) we get the correct Compton wavelength $l_\pi = r$ of the pion.
However this appears to go against the fact that there would be pair annihilation with the release of two photons. Nevertheless if we consider discrete space time, the situation would be different. In this case the Schrodinger equation

$$H\psi = E\psi \tag{16}$$

where H contains the above Coulumb interaction could be written, in terms of the space and time separated wave function components as (Cf. also ref.[21]),

$$H\psi = E\phi T = \phi\imath\hbar[\frac{T(t-\tau) - T}{\tau}] \tag{17}$$

where τ is the minimum time cut off which in the above work has been taken to be the Compton time. If, as usual we let $T = exp(irt)$ we get

$$E = -\frac{2\hbar}{\tau} sin\frac{\tau r}{2} \tag{18}$$

(18) shows that if,

$$|E| < \frac{2\hbar}{\tau} \tag{19}$$

holds then there are stable bound states. Indeed inequality (19) holds good when τ is the Compton time and E is the total energy mc^2. Even if inequality (19) is reversed, there are decaying states which are relatively stable around the cut off energy $\frac{2\hbar}{\tau}$.
This is the explanation for treating the pion as a bound state of an electron and a positron, as indeed is borne out by its decay mode. The situation is similar to the case of Bohr orbits– there also the electrons would according to classical ideas have collapsed into the nucleus and the atoms would have disappeared. In this case it is the discrete nature of space time which enables the pion to be a bound state as described by (15).
Another imprint of discrete space time can be found in the Kaon decay puzzle, as pointed out by the author[22]. There also we have equations like (16) and (17) above, with the energy term being given by $E(1+i)$, due to the fact that space time is quantized. Not only is the fact that the imaginary and real parts of the energy are of the same order borne out but as pointed out in[22] this also explains the recently observed [23] Kaon decay and violation of the

time reversal symmetry. In the words of Penrose[24], "the tiny fact of an almost completely hidden time-asymmetry seems genuinely to be present in the K^0-decay. It is hard to believe that nature is not, so to speak, trying to tell something through the results of this delicate and beautiful experiment."
From an intuitive point of view, the above should not be surprising because time or even space reversal symmetry is based on a space time continuum and is no longer obvious if space time were discrete.
Indeed this can be seen from equation (8): If we retain terms $\sim l^2$ then there is no invariance under not just time but also space reflections. It is within this framework that we can also explain the handedness of the nearly massless neutrino: It has a comparatively large Compton wavelength l and by (8) space reflection symmetry no longer holds.

References

[1] Goldstein, H., 1966 "Classical Mechanics", Addison-Wesley, Reading, Mass., p76.

[2] Misner, C.W., Thorne, K.S., and Wheeler, J.A., 1973 "Gravitation", W.H. Freeman, San Francisco, p883.

[3] Sidharth, B.G., 2000 "Comment on the paper... " Chaos, Solitons and Fractals, 21(41) L25-26.

[4] Rosen, N., 1993 International Journal of Theoretical Physics, 32 (8), 1435-1440.

[5] Huang, K., 1975 "Statistical Mechanics", Wiley Eastern, New Delhi,pp230-237.

[6] Sidharth, B.G., 1999 in Proceedings of Eighth Marcel Grossmann Meeting on General Relativity, T. Piran and Remo Ruffini (Eds.), World Scientific, Singapore,pp479-481.

[7] Sidharth, B.G., 2000 Comment on the paper..." Chaos, Solitons and Fractals, 21(41) L25-26.

[8] Newman, E.T., 1973 J.Math.Phys.14 (1) p102.

[9] Sidharth, B.G., 1998 Int.J.Mod.Phys.A., 13(15), pp2599-2612.

[10] Dirac, P.A.M., 1958 "Principles of Quantum Mechanics", Clarendon Press, Oxford, p263.

[11] Sidharth, B.G., 1997 Ind.J. of Pure and Applied Phys., 35, pp456-471.

[12] Snyder, H.S., 1947 Phys. Rev. **71** (1), 38ff.

[13] Sidharth, B.G., 2000 Chaos, Solitons and Fractals, 11 (8), p1269-1278.

[14] Ne'eman, Y., 1999 in Proceedings of Frontiers of Fundamental Physics, (Eds.) B.G. Sidharth and A. Burinskii, Universities Press, Hyderabad, pp83-96.

[15] Sidharth, B.G., "Issues in Quantized Fractal Space Time" to appear in Chaos, Solitons and Fractals.

[16] Evans, M.W., 1997 "Origin, Observation and Consequences of the $B^{(3)}$ Field" in The Present Status of the Quantum Theory of Light, S. Jeffers et al. (eds), Kluwer Academic Publishers, Netherlands, pp117-125 and several other references therein.

[17] Feynman, R.P., 1965 "The Feynman Lectures on Physics", Vol II, Addison-Wesley, Mass, p15-10-15-13.

[18] Sidharth, B.G., and Lobanov, Yu Yu, 1999 Proceedings of Frontiers of Fundamental Physics, (Eds.) B.G. Sidharth and A. Burinskii, Universities Press, Hyderabad, pp68-74.

[19] Sidharth, B.G., 1999 Mod.Phys.Lett.A., Vol.14 No.5, pp387-389.

[20] Sidharth, B.G., 1997 Mod.Phys.Lett.A., Vol.12 No.32, pp2469-2471.

[21] Wolf C., 1994 Nuovo.Cim.B 109 (3), 213.

[22] Sidharth, B.G., 2000 Chaos, Solitons and Fractals, 11, 1045-1046.

[23] Angelopoulos, A., et al., 1998 Phys.Lett.B 444, 43.

[24] Hawking, S., Israel, W., 1979 General Relativity: An Einstein Centenary Survey, Cambridge University Press, Cambridge, pp581-638.